

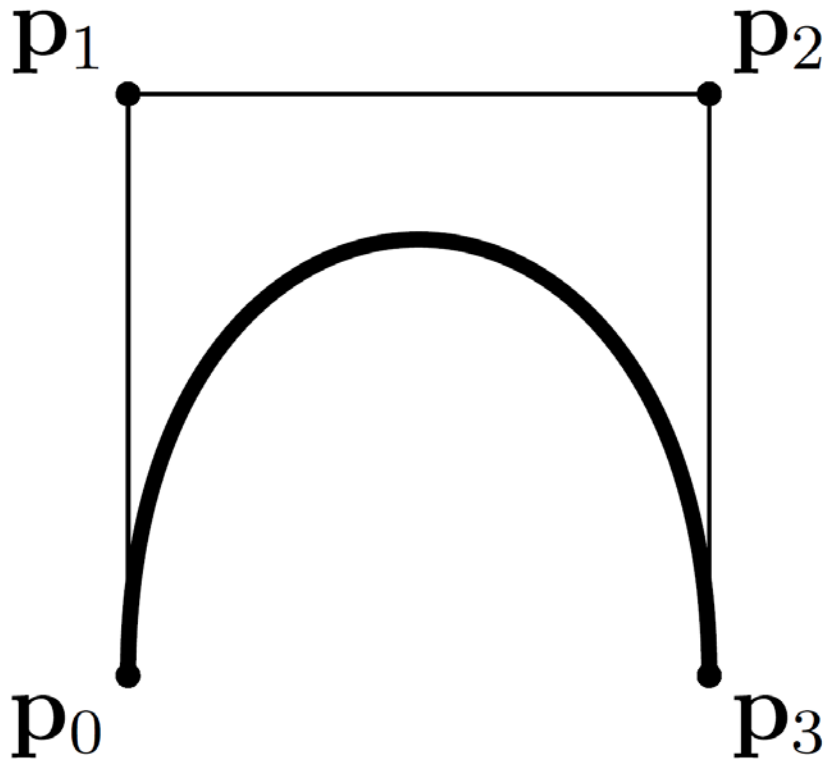
B-Splines

CS 418

Interactive Computer Graphics

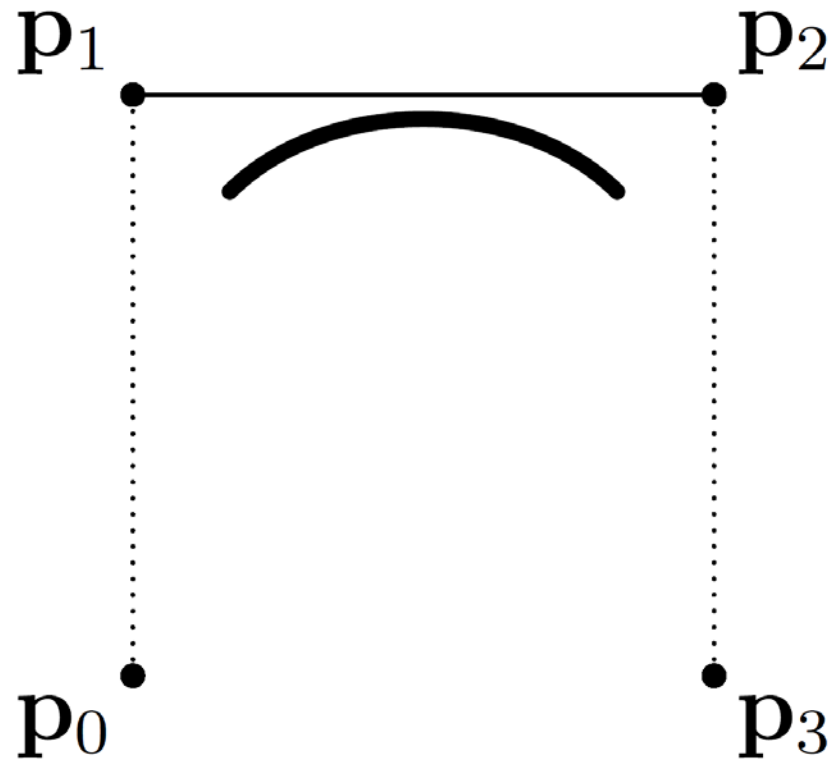
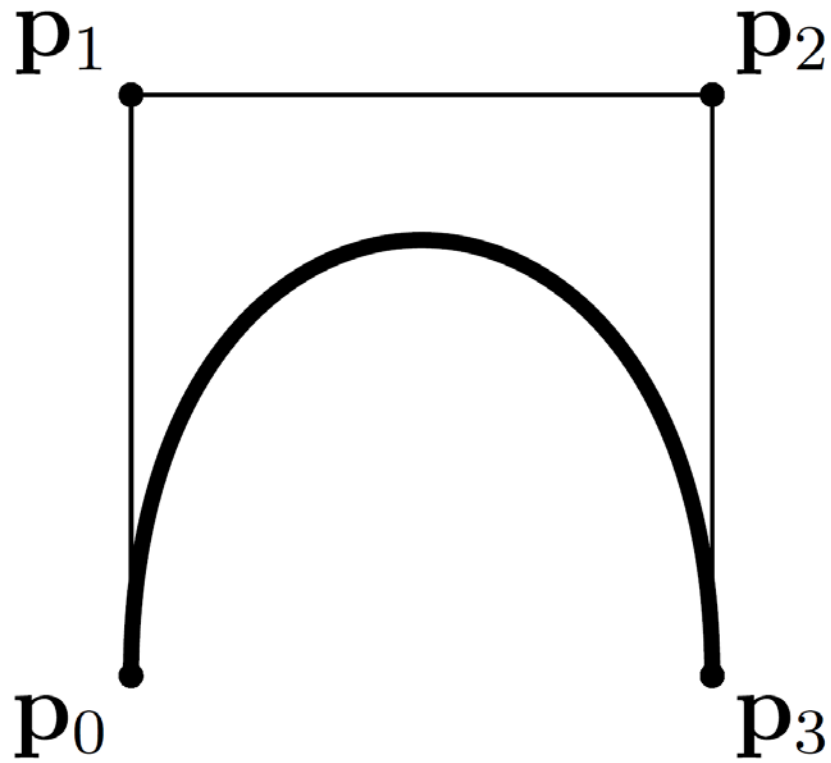
John C. Hart

Bezier Curve

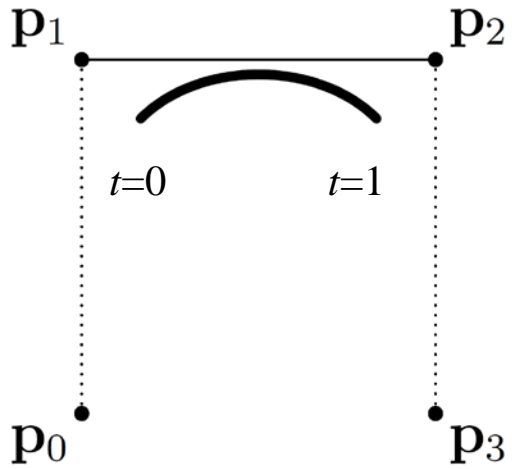


Bezier Curve

v. B-Spline

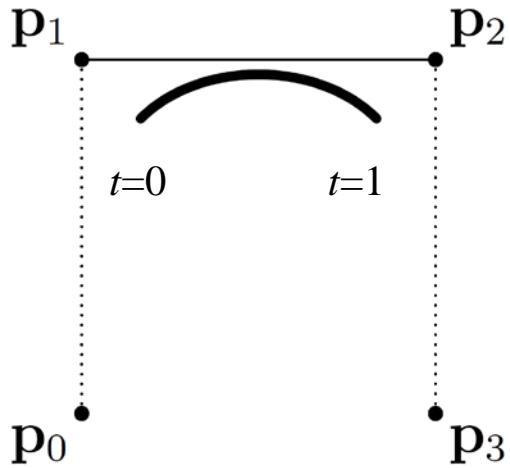


B-Spline Segment



$$\begin{aligned} \mathbf{p}(t) = & (-1/6 \mathbf{p}_0 + 1/2 \mathbf{p}_1 - 1/2 \mathbf{p}_2 + 1/6 \mathbf{p}_3) t^3 + \\ & (1/2 \mathbf{p}_0 - \mathbf{p}_1 + 1/2 \mathbf{p}_2) t^2 + \\ & (-1/2 \mathbf{p}_0 + 1/2 \mathbf{p}_2) t + \\ & 1/6 \mathbf{p}_0 + 2/3 \mathbf{p}_1 + 1/6 \mathbf{p}_2 \end{aligned}$$

B-Spline Segment

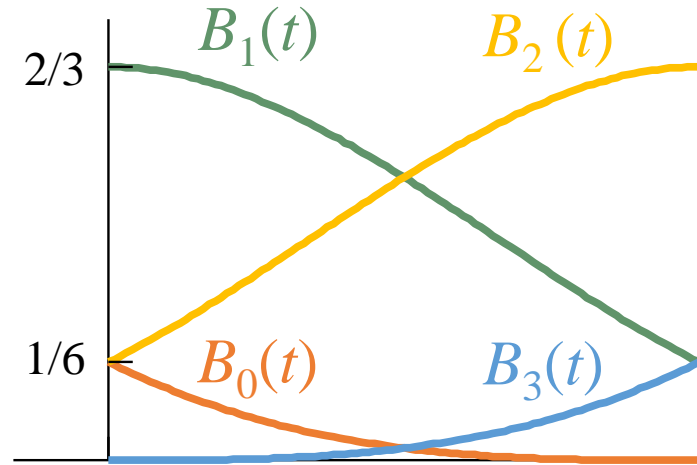
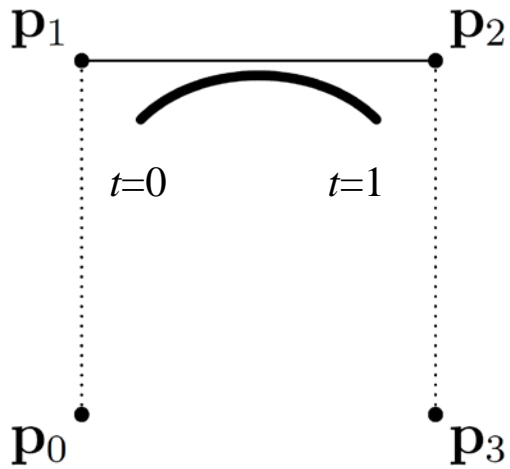


$$\begin{aligned} \mathbf{p}(t) = & (-1/6 \mathbf{p}_0 + 1/2 \mathbf{p}_1 - 1/2 \mathbf{p}_2 + 1/6 \mathbf{p}_3) t^3 + \\ & (1/2 \mathbf{p}_0 - \mathbf{p}_1 + 1/2 \mathbf{p}_2) t^2 + \\ & (-1/2 \mathbf{p}_0 + 1/2 \mathbf{p}_2) t + \\ & 1/6 \mathbf{p}_0 + 2/3 \mathbf{p}_1 + 1/6 \mathbf{p}_2 \end{aligned}$$

$$\begin{aligned} \mathbf{p}(t) = & (-1/6 t^3 + 1/2 t^2 - 1/2 t + 1/6) \mathbf{p}_0 + \\ & (1/2 t^3 - t^2 + 2/3) \mathbf{p}_1 + \\ & (-1/2 t^3 + 1/2 t^2 + 1/2 t + 1/6) \mathbf{p}_2 + \\ & (1/6 t^3) \mathbf{p}_3 \end{aligned}$$

$$= B_0(t) \mathbf{p}_0 + B_1(t) \mathbf{p}_1 + B_2(t) \mathbf{p}_2 + B_3(t) \mathbf{p}_3$$

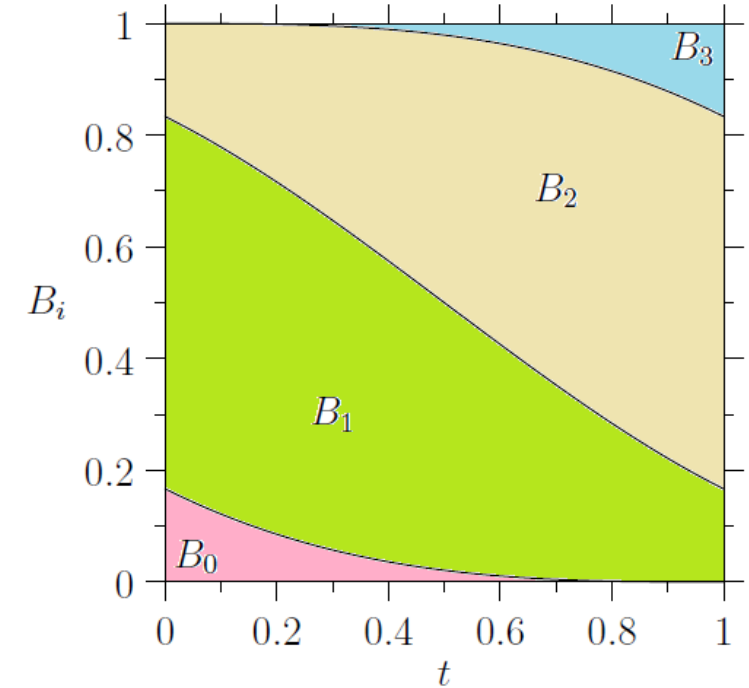
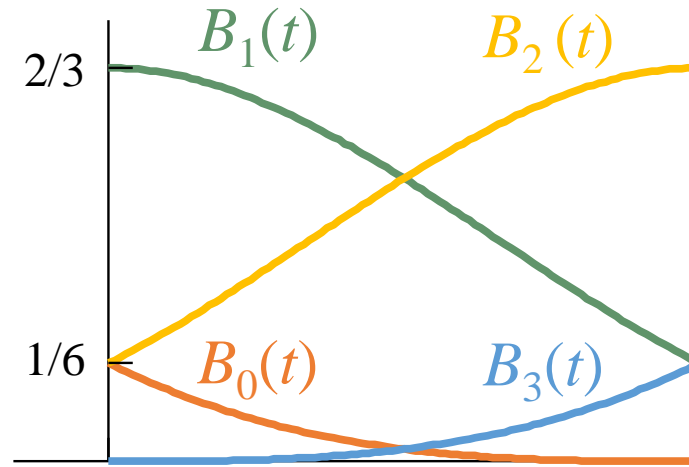
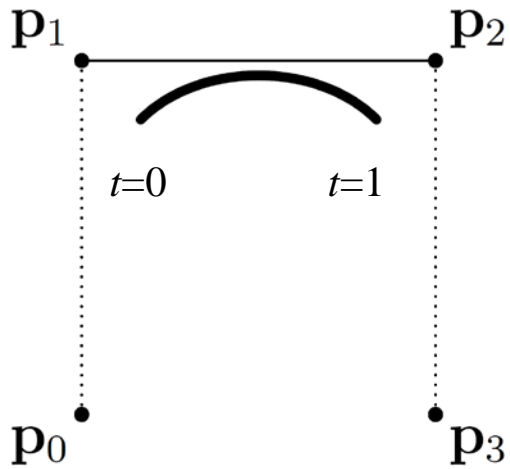
B-Spline Basis



$$\begin{aligned} \mathbf{p}(t) = & (-1/6t^3 + 1/2t^2 - 1/2t + 1/6)\mathbf{p}_0 + \\ & (1/2t^3 - t^2 + 2/3)\mathbf{p}_1 + \\ & (-1/2t^3 + 1/2t^2 + 1/2t + 1/6)\mathbf{p}_2 + \\ & (1/6t^3)\mathbf{p}_3 \end{aligned}$$

$$= B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1 + B_2(t)\mathbf{p}_2 + B_3(t)\mathbf{p}_3$$

Partition of Unity



$$\begin{aligned} \mathbf{p}(t) = & (-1/6t^3 + 1/2t^2 - 1/2t + 1/6)\mathbf{p}_0 + \\ & (1/2t^3 - t^2 + 2/3)\mathbf{p}_1 + \\ & (-1/2t^3 + 1/2t^2 + 1/2t + 1/6)\mathbf{p}_2 + \\ & (1/6t^3)\mathbf{p}_3 \end{aligned}$$

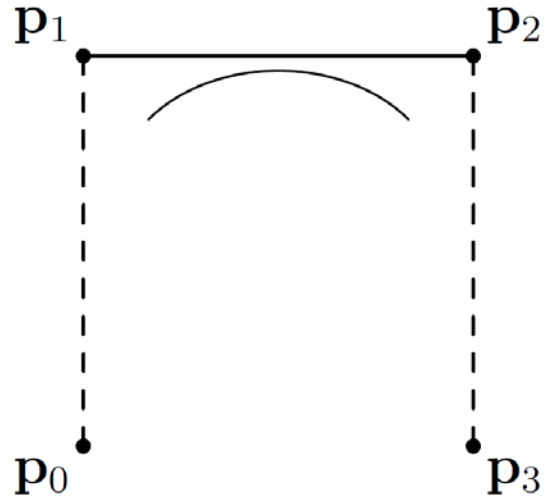
$$= B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1 + B_2(t)\mathbf{p}_2 + B_3(t)\mathbf{p}_3$$

$$B_0(t) + B_1(t) + B_2(t) + B_3(t) = 1$$

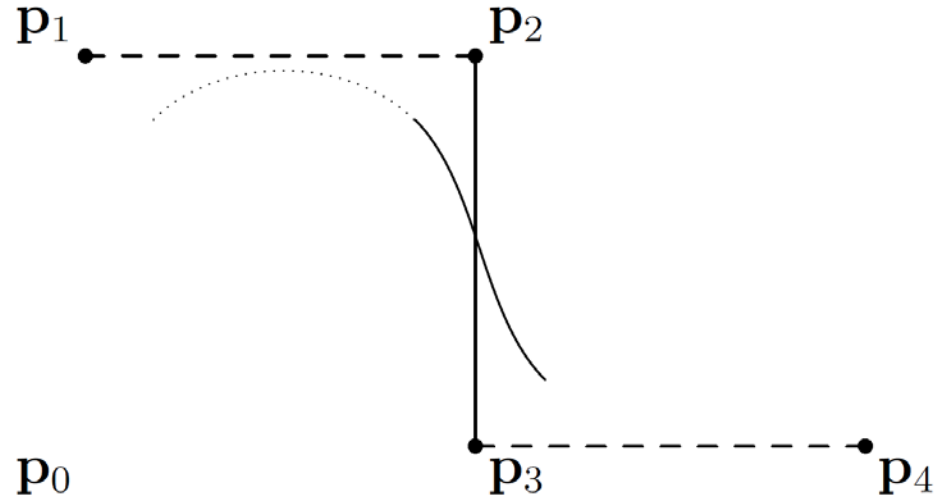
$$B_i(t) \geq 0$$

B-spline curve always in convex hull of control points

Continuity

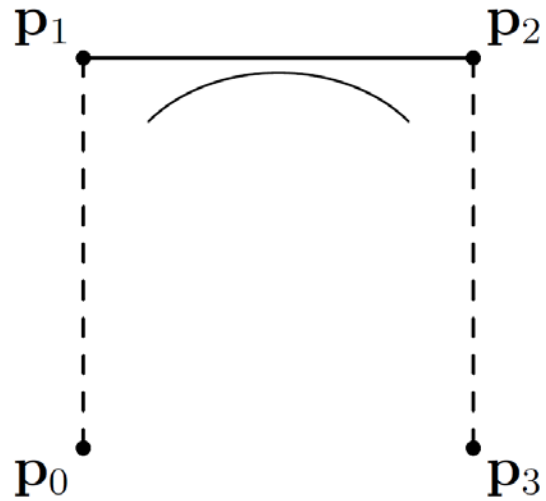


$$\mathbf{p}_{0123}(t) = B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1 + B_2(t)\mathbf{p}_2 + B_3(t)\mathbf{p}_3$$

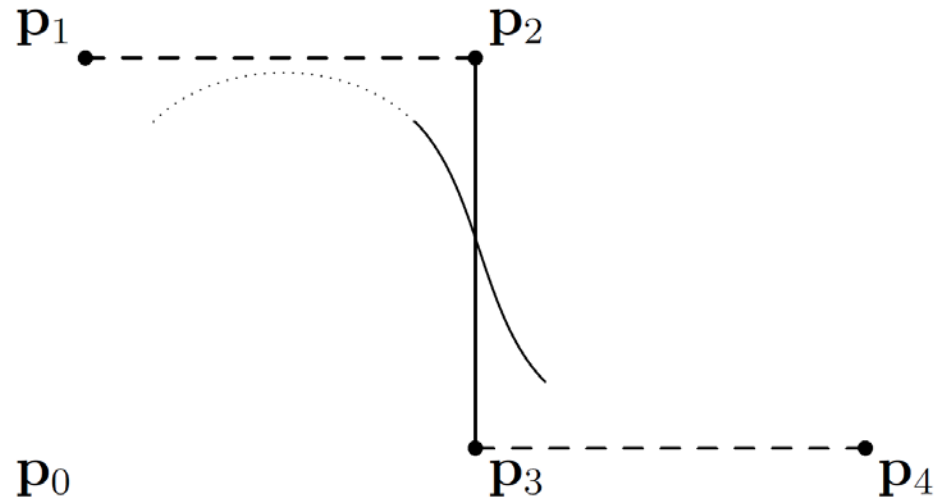


$$\mathbf{p}_{1234}(t) = B_0(t)\mathbf{p}_1 + B_1(t)\mathbf{p}_2 + B_2(t)\mathbf{p}_3 + B_3(t)\mathbf{p}_4$$

Continuity



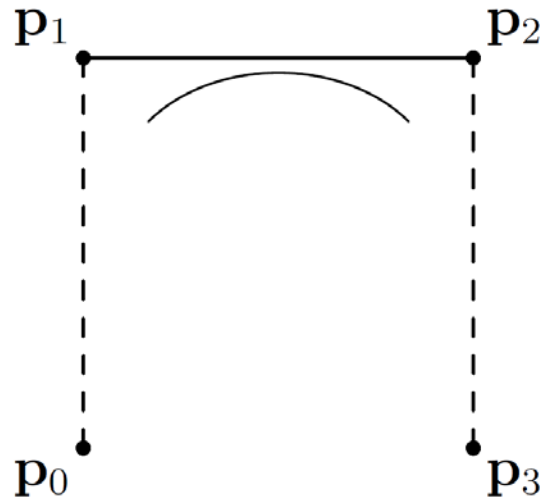
$$\mathbf{p}_{0123}(t) = B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1 + B_2(t)\mathbf{p}_2 + B_3(t)\mathbf{p}_3$$



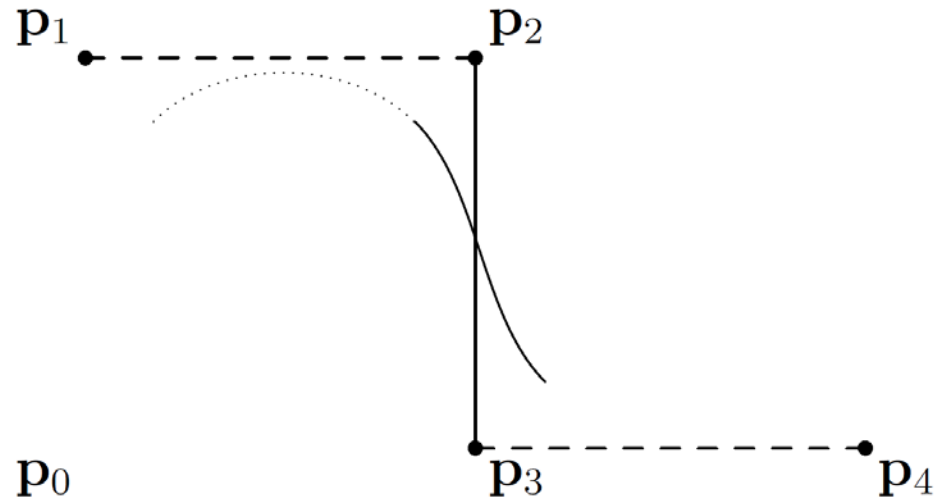
$$\mathbf{p}_{1234}(t) = B_0(t)\mathbf{p}_1 + B_1(t)\mathbf{p}_2 + B_2(t)\mathbf{p}_3 + B_3(t)\mathbf{p}_4$$

$$\mathbf{p}_{0123}(1) = \mathbf{p}_{0123}(0)$$

Continuity



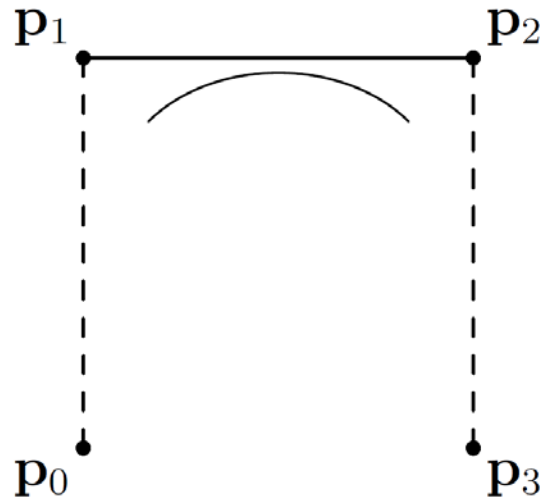
$$\mathbf{p}_{0123}(t) = B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1 + B_2(t)\mathbf{p}_2 + B_3(t)\mathbf{p}_3$$



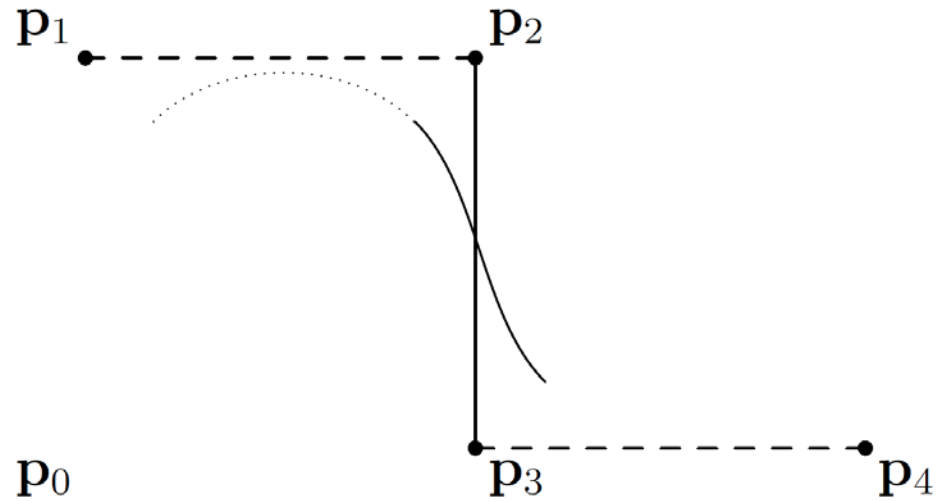
$$\mathbf{p}_{1234}(t) = B_0(t)\mathbf{p}_1 + B_1(t)\mathbf{p}_2 + B_2(t)\mathbf{p}_3 + B_3(t)\mathbf{p}_4$$

$$\begin{aligned}\mathbf{p}_{0123}(1) &= \mathbf{p}_{0123}(0) \\ \mathbf{p}'_{0123}(1) &= \mathbf{p}'_{0123}(0)\end{aligned}$$

Continuity



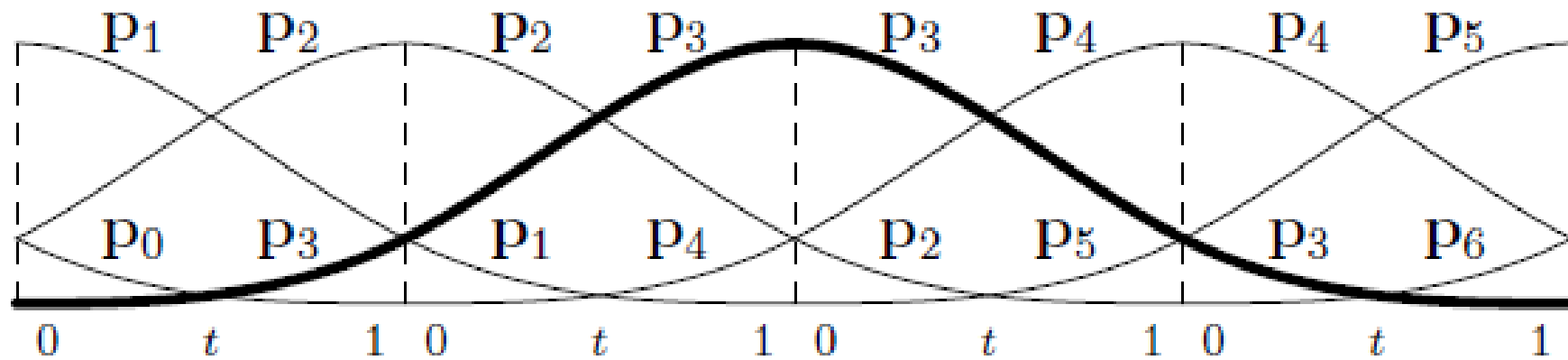
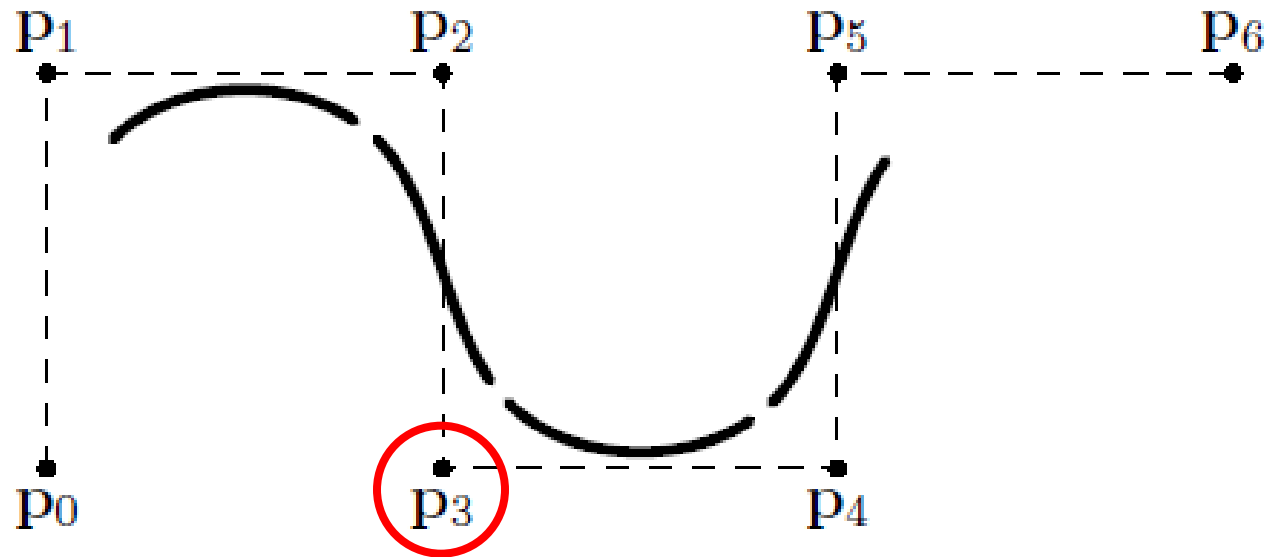
$$\mathbf{p}_{0123}(t) = B_0(t)\mathbf{p}_0 + B_1(t)\mathbf{p}_1 + B_2(t)\mathbf{p}_2 + B_3(t)\mathbf{p}_3$$



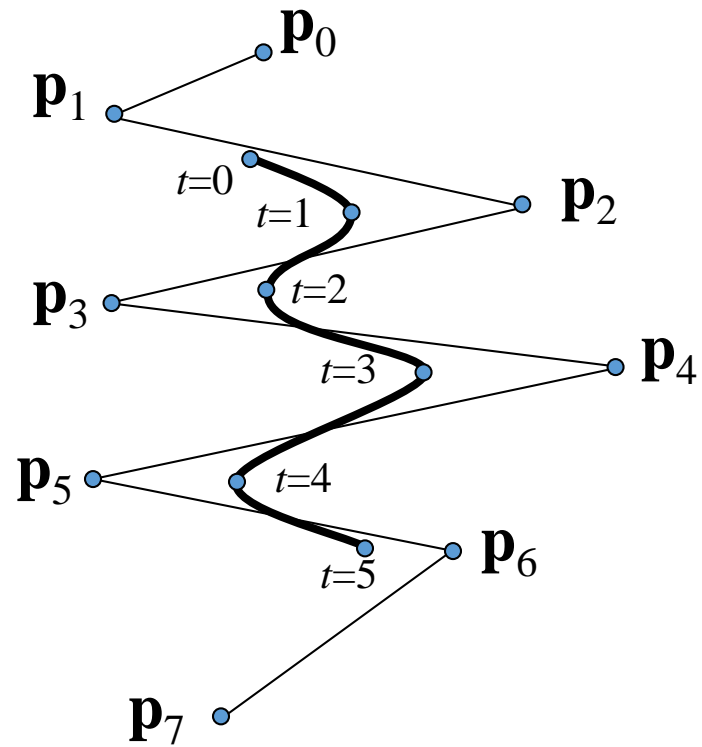
$$\mathbf{p}_{1234}(t) = B_0(t)\mathbf{p}_1 + B_1(t)\mathbf{p}_2 + B_2(t)\mathbf{p}_3 + B_3(t)\mathbf{p}_4$$

$$\begin{aligned}\mathbf{p}_{0123}(1) &= \mathbf{p}_{0123}(0) \\ \mathbf{p}'_{0123}(1) &= \mathbf{p}'_{0123}(0) \\ \mathbf{p}''_{0123}(1) &= \mathbf{p}''_{0123}(0)\end{aligned}$$

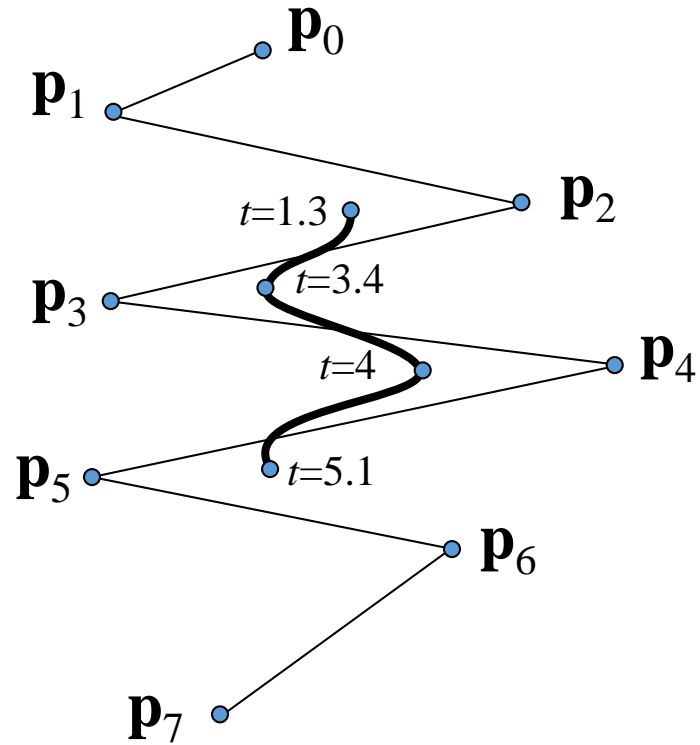
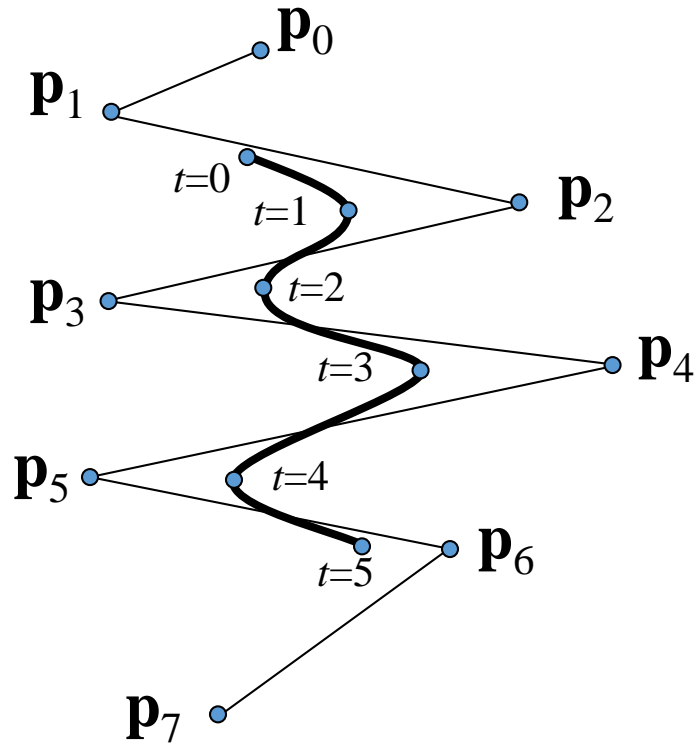
Joining B-Splines



Uniform B-Splines



Uniform v. Non-Uniform B-Splines



knot vector: $[0, .5, 1.3, 3.4, 4, 5.1, 6, 7]$